Parametric Evaluation of the Inter-Cell Interference in Coexisted Long Term Evolution Networks

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***Abstract*—In this paper, we study the effect of cochannel interference (CCI) on the performance of partially coherent BPSK and QPSK in uncorrelated L-branch equal-gain combining systems. We consider a generalized propagation model wherein the desired and interfering signals undergo Nakagami-m or Rician fading with different amounts of fading severity. Further, the interfering signals are assumed to be asynchronous symbol timing with the desired signal, so that the effect of cross-signal intersymbol interference (ISI) is taken into account. Using a convergent Fourier series method, we derive extensive analytical results for the average bit error probability and the SNR gain penalty caused by the interference signals for different signal-to-interference ratio levels. The numerical results presented in this paper demonstrate the system performance under very realistic propagation and detection conditions including CCI, carrier phase error recovery, cross-signal ISI, generalized fading channels, and AWGN. Hence our results are expected to be of significant practical use for such scenarios.**

***Index Terms*—Cochannel interference, intersymbol interference, generalized fading channels, carrier phase error, quazi-analytical simulation, equal-gain combining.**

I. Introduction

SINCE the radio spectrum is scarce and is a very precious commodity, the same sets of carrier frequency are reused in cells spatially separated by a predetermined reuse distance from the subject cell of interest to achieve maximum spectral efficiency. Therefore, a desired mobile user signal is corrupted by the interference generated by other undesired active users' signals in the neighboring cells operating at the same carrier frequency. This results in the so called cochannel interference (CCI), a problem categorized as the bottleneck for the capacity of any wireless cellular system. On the other hand, due to the time-variant characteristic of wireless channel, the carrier phase cannot be recovered perfectly (i.e., partially coherent detection), which results in system performance degradation [1]. It is well-known that diversity combining is a powerful technique for combating the negative effects of wireless channel impairments and is often employed in practice. Therefore, it is of great interest to investigate the effect of CCI on the performance of partially coherent fading systems, and to examine the ability of diversity combining as a means for combating all these impairments (CCI, multipath fading, carrier phase error, etc.) together.

Different CCI environments have been proposed to characterize the propagation of the desired and interfering signals. In early analysis [2], both the desired and the interfering user signals are modeled by Rayleigh fading. Although the Rayleigh fading model is widely used to represent propagation environments when no line-of-sight signal exists between the transmitter and the receiver, some propagation environments might not be well characterized by this fading model. For example, in microcellular environments, the fading is not as much severe as Rayleigh fading and it is better to be characterized by either Nakagami-m or Rician fading. Therefore, a Nakagami-m/Nakagami-m model or Rician/Rician model, in which the desired user signal is described by Nakagami-m or Rician fading and the interfering signals are described by different Nakagami-m or Rician fading levels are general enough models to cover a wide range of real CCI fading conditions [3]. On the other hand, in practical systems, the interfering signals are usually not symbol synchronous with the desired signal. This is simply because those signals are in general propagating through different paths that have different lengths, phases, and time delays. Hence, it is more practically appropriate to assume that the interfering signals are timing asynchronous with the desired signal. In this study, a general scenario wherein both Nakagami-m/Nakagami-m and Rician/Rician CCI models under asynchronous or synchronous operations is considered to evaluate the performance of partially coherent BPSK and QPSK wireless systems with diversity reception.

Many papers dealt with the error rate performance of PSK systems in flat frequency-nonselective fading and CCI. The bit error rate performance of single channel (no diversity) coherent PSK-faded systems can be found in [2]-[4]. The bit error probability (BEP) of coherent BPSK systems in Rayleigh/Rayleigh desired/interferering signals is derived in [2], [5]. Fourier series approximation is used in [6] to analyze the performance of QPSK system in Nakagami-m/Rayleigh and Ricain/Rayleigh CCI asynchronous models. The authors in [7] derived closed-form solution for the average BEP of BPSK in synchronous Nakagami-m/Nakagami-m CCI. However, the solution in [7] is reached under the assumption that the sum of the Nakagami-m interfering signals is a Gaussian random variable (RV), which is still an approximation. Recently, the exact BEP of asynchronous interferers with nonreturn-to-zero (NRZ) pulse shape signaling are derived in [8] for BPSK in Rayleigh/Rayleigh CCI and in [4] for BPSK in Nakagami-

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m/Nakagami-m or QPSK in Nakagami-m/Rayleigh CCI models. More generally, the error rate performance for BPSK with several bandwidth-efficient pulse shape signaling in Nakagami-m/Nakagami-m asynchronous CCI is studied in [3]. The analyses in the last three studies are based on the characteristic function (CF) inversion method, where the final error rate expression is given in terms of infinite-range integral that has to be solved using numerical techniques. For diversity systems, the BEP for coherent BPSK was studied for dual-branch system with asynchronous Nakagami-m or Rician/Rayleigh CCI in [9] and for L-branch EGC with synchronous multiple Rician CCI in [10]. However, in all previous studies, a hard to accomplish perfect coherent detection of the carrier phase is assumed. To the author's knowledge, the performance of L-branch partially coherent BPSK and QPSK fading systems in the presence of asynchronous or synchronous CCI has not been investigated so far.

The paper is organized as follows. In Section II, we describe our system model. In Section III, Fourier infinite series approach is used to derive expressions for the average BEP of partially coherent BPSK and QPSK in equal-gain combining (EGC) reception and in the presence of CCI. We consider both Nakagami-m/Nakagami-m and Rician/Rician models with asynchronous or synchronous interferers and NRZ pulse-shape filtering. The analytical expressions obtained are evaluated numerically and discussed in Section IV. Finally, we make our concluding remarks in Section V.

II. System Description

A. Signals and Receiver Structure

We consider coherent PSK systems (both BPSK and QPSK) with cochannel interference in frequency-nonselective slowly fading environment. The complex baseband representation of the desired transmitted PSK signal is given by

$$s_d(t) = \sqrt{2P_d} \sum_{k=-\infty}^{\infty} e^{j\theta_d(k)} h_T(t-kT), \tag{1}$$

where, for the desired user's signal, P_d is the transmitted symbol power and $\theta_d(k)$ represents the information phase over the kth signaling period interval. $\theta_d(k)$ takes values from $\{0, \pi\}$ radians in BPSK scheme and from $\{-3\pi/4, -\pi/4, \pi/4, 3\pi/4\}$ radians in QPSK scheme with equal probabilities. The impulse response of the transmitter filter, $h_T(t)$, is assumed to be constant over the symbol time period T (i.e., NRZ pulse shaping) and its energy is normalized to 1, i.e., $h_T(t) = \frac{1}{\sqrt{T}}, \quad 0 \le t \le T$. As the desired signal propagates over wireless channel, it suffers from multipath fading as well as interference from other user's signals using the same band of frequency. If we reasonably assume that the desired and the interfering signals have the same modulation scheme, then we write the transmitted signal of the ith interferer as

$$s_i(t) = \sqrt{2P_i} \sum_{k=-\infty}^{\infty} e^{j\theta_i(k)} h_T(t-kT), \tag{2}$$

where P_i, $\theta_i(k)$ are, respectively, the transmitted power and the information phase of the ith interfering signal.

Let us consider a general propagation scenario in which the desired signal is corrupted by I interfering signals and is being received over L-branch diversity system. For slow frequency-nonselective fading, the total received signal over the lth diversity branch can be written as

$$r_l(t) = g_{dl} s_d(t) + \sum_{i=1}^{I} g_{il} s_i(t-\tau_i) + n_l(t), \quad l = 1, 2, \cdots, L \tag{3}$$

where $g_{dl} = \alpha_{dl} e^{j\phi_{dl}}$ and $g_{il} = \alpha_{il} e^{j\phi_{il}}$ are, respectively, the desired and the ith interfering user's complex channel fading gains. The channel fading envelopes (α_{dl}, α_{il}) and the fading phases (ϕ_{dl}, ϕ_{il}) are RVs assumed to be mutually independent from branch to branch and independent of the background complex-valued AWGN processes $n_l(t)$. In (3), the possible symbol timing offset between the ith interferer and the desired user is represented by τ_i, which is assumed to be identical for all diversity paths (this is reasonable since the I interfering signals could be received from the same constellation scatters) and uniformly distributed over $[0, T)$.

Since EGC has performance close to the optimal maximum ratio combining with much simpler implementation, we study the impact of CCI on EGC. In EGC technique, each diversity signal is passed through a receiver filter $h_R(t)$ that is matched to the transmit filter $h_T(t)$ (i.e., $h_R(t) = h_T^*(t) = 1/\sqrt{T}$), co-phased with respect to the desired user phase, ϕ_{dl}, sampled properly at the symbol times $t_k = kT$, and finally summed to produce the combiner's decision variable. In practical coherent systems, fading and interference make it difficult to perform perfect phase tracking. Hence, we consider the general case when the carrier phase of the desired user, ϕ_{dl}, is imperfectly recovered. Therefore, without loss of generality, if the zeroth data symbol, $\theta_d(0)$, is to be detected, the combiner decision statistic can be written as

$$\begin{aligned} U[\theta_d(0)] &= \sum_{l=1}^{L} e^{-j\hat{\phi}_{dl}} \int_0^T r_l(t) h_R(t)\, dt|_{t=0} \\ &= \sum_{l=1}^{L} \Bigg[\sqrt{2P_d} \alpha_{dl} e^{j[\epsilon_l + \theta_d(0)]} + \sum_{i=1}^{I} \sqrt{2P_i} \\ &\qquad \times \alpha_{il} e^{j\hat{\phi}_{il}} \Lambda(\theta_i, \tau_i) \Bigg] + n_I(0) + jn_Q(0), \end{aligned} \tag{4}$$

where $\epsilon_l \triangleq \phi_{dl} - \hat{\phi}_{dl}$ is the desired user introduced carrier phase error at the lth branch assumed to follow Tikhonov distribution [1, eqn. 4] with rms phase error $\sigma_{\epsilon l}$. A typical values of $\sigma_{\epsilon l}$ ranges from 5° to 20° corresponding to a loop SNR, ρ_{cl}, of (21.2-9.4) dBs. However, in some cellular applications, larger phase error variations due to nasty severer fading conditions may occur, and hence, noncoherent or differentially coherent systems could become attractive alternatives to partially coherent systems with receiver simplicity [11]. On the other hand, the possible random phase difference between the ith interfering user's fading phase and the estimate of the desired user fading phase denoted by $\hat{\phi}_{il} \triangleq \phi_{il} - \hat{\phi}_{dl}$ is reasonably assumed to be uniformly-distributed over $[0, 2\pi)$, since no attempt has been made to estimate ϕ_{il}. Again, $n_I(0)$ and $n_Q(0)$ are the filtered Gaussian variables with zero means

and equal variances of $L\sigma^2$. In (4), and for the lth path, the first term inside the square brackets represents the desired signal component while the sum term represents the undesired CCI component. Moreover, due to the timing offsets τ_i's, each CCI signal will be subject to cross-signal ISI component as will be indicated by $\Lambda(\theta_i, \tau_i)$, which can be written as

$$\begin{aligned}\Lambda(\theta_i, \tau_i) &= \frac{1}{\sqrt{2P_i}} \int_0^T s_i(t-\tau_i) h_R(t) dt \\ &= e^{j\theta_i(0)} \lambda_0(\tau_i) + e^{j\theta_i(-1)} \lambda_{-1}(\tau_i),\end{aligned} \tag{5}$$

where $\theta_i(0)$ and $\theta_i(-1)$ indicate the two consecutive phases of the ith interfering signal overlapping with the desired phase $\theta_d(0)$, $\lambda_0(\tau_i) = \int_{\tau_i}^{T} 1/T dt = 1 - \tau_i/T$ represents the contribution of $\theta_i(0)$, and $\lambda_{-1}(\tau_i) = \lambda_0(T - \tau_i) = \tau_i/T$ represents the contribution of the ISI phase $\theta_i(-1)$. Note that for synchronized interferers scenario, i.e., $\{\tau_i = 0\}$, $\Lambda(\theta_i) = e^{j\theta_i(0)}$ and no cross-signal ISI component exists.

B. CCI Analysis Model

We consider a general propagation model wherein the desired and interfering signals allow to follow Nakagami-m or Rician fading with different levels of fading severity. Thus, the desired user's fading envelope over the lth channel, α_{dl}, follows either Nakagami-m distribution [1, eqn. 3] with fading severity parameter $m_{dl} \geq 0.5$ and average fading power $\Omega_{dl} \triangleq E[\alpha_{dl}^2]$, or Rician distribution [1, eqn. 2] with fading figure $K_{dl} \geq 0$ and average power Ω_{dl}. Similarly, we assume that the interfering user's fading envelopes over the lth channel, α_{il}, are either Nakagami-m-distributed with parameters (m_{il}, Ω_{il}) or Rician-distributed with parameters (K_{il}, Ω_{il}). Those fading and interference models are general enough to cover a wide range of realistic conditions.

Without loss of generality, we assume that the average symbol transmitted power for both the desired and all the interfering users are the same, i.e., $P_d = P_1 = P_2 = \cdots = P_I = P_b \log_2 M$, where M is the modulation order and P_b is the average bit power. Assume further that $P_b = \sigma^2$, so, with this convention, the fading SNR and signal-to-interference ratio (SIR) per bit in the lth channel for our system model can be expressed as

$$\begin{aligned}\text{SNR}_l &= \frac{P_b \Omega_{dl}}{\sigma^2} = \Omega_{dl} \\ \text{SIR}_l &= \frac{P_b \Omega_{dl}}{P_b \sum_{i=1}^{I} \Omega_{il}} = \frac{\Omega_{dl}}{\sum_{i=1}^{I} \Omega_{il}}.\end{aligned} \tag{6}$$

III. ERROR RATE ANALYSIS

In this section, we evaluate the BEP performance for the Nakagami-m/Nakagami-m and Rician/Rician - desired/interfering CCI models when partially coherent BPSK and QPSK schemes are used in wireless diversity communications.

A. BPSK Error Performance

Assuming that phase 0 radian was transmitted over the 0th signaling interval, i.e., $\theta_d(0) = 0$, and for equal probable symbols, then the average BEP for EGC BPSK system in the presence of synchronization phase error and CCI can be written using (4) and (5) as

$$\begin{aligned}P_b(e) &= \Pr[\Re\{U[\theta_d(0)]\} \leq 0 | \theta_d(0) = 0] \\ &= \Pr\Bigg[\sum_{l=1}^{L}\Bigg\{\sqrt{2P_b}\alpha_{dl}\cos\epsilon_l + \sum_{i=1}^{I}\sqrt{2P_b}\alpha_{il} \\ &\qquad \times \Re\left[e^{j\hat{\phi}_{il}}\Lambda(\theta_i,\tau_i)\right]\Bigg\} + \sqrt{L\sigma^2}N_I < 0\Bigg] \\ &= \Pr\Bigg[Z_2 = \sum_{l=1}^{L}\Bigg\{\underbrace{\alpha_{dl}\cos\epsilon_l}_{Z_{dl}} + \sum_{i=1}^{I}\underbrace{\alpha_{il}\cos\hat{\phi}_{il}\lambda_2(\theta_i,\tau_i)}_{z_{il}}\Bigg\} \\ &\qquad + \sqrt{\frac{L}{2}}N_I < 0\Bigg]\end{aligned} \tag{7}$$

where $\Re(\cdot)$ denotes the real part, N_I is the normalized AWGN variable, and

$$\lambda_2(\theta_i, \tau_i) = \lambda_0(\tau_i)\cos\theta_i(0) + \lambda_{-1}(\tau_i)\cos\theta_i(-1) \tag{8}$$

Note that in obtaining (8) we use the fact that $\sin\theta_i(0) = \sin\theta_i(-1) = 0$. To evaluate (7) analytically, we use the well-known Fourier series-based expansion for the cumulative distribution function (cdf) that has been derived by Beaulieu in the early 1990s. In particular, $P_b(e)$ can be computed within a determined accuracy using [12, eqn. 6b] as

$$P_b(e) = \frac{1}{2} - \sum_{n=0}^{\infty} \frac{2\Im\{\Phi_{Z_2}[(2n+1)\eta]\}}{\pi(2n+1)}, \quad \eta = \frac{2\pi}{T_s} \tag{9}$$

where $\Im(z)$ is the imaginary part of z, $\Phi_X(\nu) \triangleq E[e^{j\nu X}]$ is the the CF of X, and T_s is a parameter controlling the accuracy of the result. We have shown in [1] that the truncation error of the Beaulieu series can be made arbitrarily small by terminating the series at a moderate number of terms, and that the convergence rate of the series improves as the diversity order, L, increases. We also showed that the selection of T_s is not critical, but more series terms are needed in the calculations if T_s is chosen larger than the minimum value needed to obtain a desired accuracy. In all computations, the series is truncated at $n = 30, 26,$ and 23 for systems with diversity order $L = 1, 2,$ and 3, respectively, while the required number of terms reduces to about 20 terms for systems with diversity order $L \geq 4$ to obtain a good accuracy. The value of T_s was chosen to lie between 10 (for $L \geq 4$) and 20 (for $L = 1$). For the problem in our hand, one can write

$$\Phi_{Z_2}(\nu) = \Phi_{N_I}\left(\sqrt{\frac{L}{2}}\nu\right) \prod_{l=1}^{L} \left\{\Phi_{Z_{dl}}(\nu) \prod_{i=1}^{I} \Phi_{z_{il}}(\nu)\right\}, \tag{10}$$

which is true assuming that the diversity channels are statistically independent, and that both the desired user's signal Z_{dl} and all the interfering user's signals z_{il}'s in (7) experience independent fades. In (10), $\Phi_{N_I}(\nu) = e^{-\nu^2/2}$ and $\Phi_{Z_{dl}}(\nu), \Phi_{z_{il}}(\nu)$ are, respectively, the CFs of the desired and the interferer users' signals. The exact infinite integral

expression for $\Phi_{Z_{dl}}(\nu)$, which is,

$$\Phi_{Z_{dl}}(\nu) = \int_0^\infty \frac{I_0(\rho_{cl} + j\nu x)}{I_0(\rho_{cl})} p_{\alpha_l}(x) dx \qquad (11)$$

can be evaluated numerically to any degree of accuracy with a finite range of x over (0, $10\Omega_l$) as discussed in [1, Sec. II. B].

To evaluate for $\Phi_{z_{il}}(\nu) = E[e^{j\nu\alpha_{il}\cos\hat{\phi}_{il}\lambda_2}]$ (we drop the λ_2's arguments for notation simplicity), we first integrate over the distribution of $\hat{\phi}_{il}$, which is uniform over $[0, 2\pi)$, to obtain the conditional CF

$$\Phi_{z_{il}}(\nu|\alpha_{il}, \lambda_2) = \frac{1}{2\pi}\int_0^{2\pi} e^{j\nu\alpha_{il}\lambda_2\cos x} dx = J_0(\nu\alpha_{il}\lambda_2), \quad (12)$$

where $J_0(\cdot)$ is the zeroth order Bessel function of the first kind. Then, to remove the condition on α_{il}, (12) has to be averaged like

$$\Phi_{z_{il}}(\nu|\lambda_2) = \int_0^\infty p_{\alpha_{il}}(x) J_0(\nu\lambda_2 x) dx. \qquad (13)$$

If α_{il} is Nakagami-m-distributed, one can show with the use of the tabulated formula [13, eqn. 6.631.1] that the solution of (13) is

$$\Phi_{z_{il}}(\nu|\lambda_2) = {}_1F_1\left(m_{il}; 1; \frac{-\Omega_{il}\nu^2}{4m_{il}}\lambda_2^2\right), \qquad (14)$$

where ${}_1F_1(\cdot;\cdot;\cdot)$ is the confluent hypergeometric function [13, eqn. 9.120.1]. Also, for Rician-distributed α_{il}, we can show with the help of the integral formula [13, eqn. 6.633.4] that

$$\Phi_{z_{il}}(\nu|\lambda_2) = \exp\left(\frac{-\Omega_{il}\nu^2}{4(1+K_{il})}\lambda_2^2\right) J_0\left(\sqrt{\frac{K_{il}\Omega_{il}}{1+K_{il}}}\ \nu\lambda_2\right). \qquad (15)$$

The unconditional CF, i.e., $\Phi_{z_{il}}(\nu)$, might be obtained by averaging the last two equations over the distribution of λ_2. Since λ_2 variable is statistically a special case of λ_4 appearing in the next section, eqn. (17), the derivation of $\Phi_{z_{il}}(\nu)$ will be similar to the derivation of $\Phi_{y_{il}}(\nu)$ in the QPSK case and will not be presented here for the sake of briefness.

B. QPSK Error Performance

Assuming Gray code technique is used to map the pair of information bits into the four possible phases, the average BEP for equal probable symbols and symmetric I and Q modulation branches in QPSK can be shown using (4) and (5) to be

$$\begin{aligned} P_b(e) &= \Pr[\Re\{U[\theta_d(0)]\} \le 0 | \theta_d(0) = \pi/4] \\ &= \Pr\Big[\sum_{l=1}^{L}\Big\{2\sqrt{P_b}\alpha_{dl}\cos(\epsilon_l + \pi/4) + \sum_{i=1}^{I} 2\sqrt{P_b}\alpha_{il} \\ &\qquad \times \Re\left[e^{j\hat{\phi}_{il}}\Lambda(\theta_i,\tau_i)\right]\Big\} + \sqrt{L\sigma^2}N_I < 0\Big] \\ &= \Pr\Big[Z_4 = \sum_{l=1}^{L}\Big\{\underbrace{\sqrt{2}\alpha_{dl}\cos(\epsilon_l + \pi/4)}_{Y_{dl}} \\ &\qquad + \sum_{i=1}^{I}\underbrace{\alpha_{il}\lambda_4(\theta_i,\tau_i)}_{y_{il}}\Big\} + \sqrt{\frac{L}{2}}N_I < 0\Big], \end{aligned} \qquad (16)$$

where

$$\begin{aligned} \lambda_4(\theta_i,\tau_i) &= \sqrt{2}\big[\lambda_0(\tau_i)\cos(\theta_i(0) + \hat{\phi}_{il}) \\ &\qquad + \lambda_{-1}(\tau_i)\cos(\theta_i(-1) + \hat{\phi}_{il})\big] \\ &= \big[\lambda_0(\tau_i)c_0 + \lambda_{-1}(\tau_i)c_{-1}\big]\cos\hat{\phi}_{il} \\ &\qquad + \big[\lambda_0(\tau_i)d_0 + \lambda_{-1}(\tau_i)d_{-1}\big]\sin\hat{\phi}_{il}, \end{aligned} \qquad (17)$$

in which, $c_0,\ c_{-1},\ d_0$ and d_{-1} take on values of ± 1 with equal probabilities, since in QPSK scheme, $\cos\theta_i(\cdot)$ and $\sin\theta_i(\cdot)$ are either $\frac{1}{\sqrt{2}}$ or $\frac{-1}{\sqrt{2}}$.

As in the BPSK case, equation (9) together with $\Phi_{Z_4}(\nu) = \Phi_{N_I}\left(\sqrt{\frac{L}{2}}\nu\right)\prod_{l=1}^{L}[\Phi_{Y_{dl}}(\nu) \times \prod_{i=1}^{I}\Phi_{y_{il}}(\nu)]$ can be used to calculate for the error rate in (16). Note that $\Phi_{Y_{dl}}(\nu)$ can be evaluated in a similar way as shown in [1, eqns. 10, 12] using

$$\Phi_{Y_{dl}}(\nu) \approx \int_0^{10\Omega_l} \frac{I_0\left(\sqrt{\rho_{cl}^2 + j2\nu x\rho_{cl} - 2\nu^2x^2}\right)}{I_0(\rho_{cl})} p_{\alpha_l}(x) dx \qquad (18)$$

Note that for the case of perfect phase recovery, i.e. $\epsilon_l = 0$, the CF of the desired user variable ($\Phi_{Z_{dl}}(\nu)$ in BPSK case or $\Phi_{Y_{dl}}(\nu)$ in QPSK case) reduced to the CF of either Rician or Nakagami variables which is well known in the literature (see for example eqn. (21b) in [3] which represents the CF for Nakagami fading with integer m). To evaluate $\Phi_{y_{il}}(\nu) = E[e^{j\nu\alpha_{il}\lambda_4}]$, we should consider for all possible values of λ_4 as follows:

- If $c_{-1} = c_0$ and $d_{-1} = d_0$ (4 possible combinations), then $\lambda_4 = \pm\cos\hat{\phi}_{il} \pm \sin\hat{\phi}_{il}$. If we denote $\Phi_{y_{il}}(\nu)$ by $\Phi_{y_{il}}^{(1)}(\nu)$, one can show with the help of [13, eq. 3.937.2] that

$$\begin{aligned} \Phi_{y_{il}}^{(1)}(\nu) &= \int_0^\infty p_{\alpha_{il}}(x)\frac{1}{2\pi}\int_0^{2\pi} e^{j\nu x(\pm\cos\theta \pm \sin\theta)} d\theta \\ &= \int_0^\infty p_{\alpha_{il}}(x) J_0(\sqrt{2}\nu x) dx, \end{aligned} \qquad (19)$$

which is for the fading distributions under consideration

can be written as

$$\Phi^{(1)}_{y_{il}}(\nu) = \begin{cases} {}_1F_1\left(m_{il}; 1; \frac{-\Omega_{il}\nu^2}{2m_{il}}\right) \\ \qquad \text{for Nakagami-}m\text{ fading} \\ \exp\left(\frac{-\Omega_{il}\nu^2}{2(1+K_{il})}\right) J_0\left(\sqrt{\frac{2K_{il}\Omega_{il}}{1+K_{il}}}\ \nu\right), \\ \qquad \text{for Rician fading} \end{cases} \tag{20}$$

- If $c_{-1} \neq c_0$ and $d_{-1} \neq d_0$ (4 possible combinations), then $\lambda_4 = (\cos\hat{\phi}_{il} + \sin\hat{\phi}_{il})u$, where u is uniformly-distributed over $[-1, 1]$. Let us refer to $\Phi_{y_{il}}(\nu)$ by $\Phi^{(2)}_{y_{il}}(\nu)$, then analogous to the previous case

$$\Phi^{(2)}_{y_{il}}(\nu|u) = \begin{cases} {}_1F_1\left(m_{il}; 1; \frac{-\Omega_{il}\nu^2}{2m_{il}}u^2\right) \\ \qquad \text{for Nakagami-}m\text{ fading} \\ \exp\left(\frac{-\Omega_{il}\nu^2}{2(1+K_{il})}u^2\right) J_0\left(\sqrt{\frac{2K_{il}\Omega_{il}}{1+K_{il}}}\nu\ u\right), \\ \qquad \text{for Rician fading} \end{cases} \tag{21}$$

and the unconditional CF can be evaluated as

$$\Phi^{(2)}_{y_{il}}(\nu) = \begin{cases} {}_2F_2\left(\frac{1}{2}, m_{il}; \frac{3}{2}, 1; \frac{-\Omega_{il}\nu^2}{2m_{il}}\right) \\ \qquad \text{for Nakagami-}m\text{ fading} \\ \int\limits_0^1 \exp\left(\frac{-\Omega_{il}\nu^2}{2(1+K_{il})}x^2\right) J_0\left(\sqrt{\frac{2K_{il}\Omega_{il}}{1+K_{il}}}\nu\ x\right) dx, \\ \qquad \text{for Rician fading} \end{cases} \tag{22}$$

- Finally, if $c_{-1} \neq c_0$ and $d_{-1} = d_0$, or $c_{-1} = c_0$ and $d_{-1} \neq d_0$ (8 possible combinations), then from (17), $\lambda_4 = \pm\cos\hat{\phi}_{il}\ u \pm \sin\hat{\phi}_{il}$, where u is uniform over $[-1, 1]$. If we mark $\Phi_{y_{il}}(\nu)$ by $\Phi^{(3)}_{y_{il}}(\nu)$, one can show that

$$\begin{aligned} \Phi^{(3)}_{y_{il}}(\nu|u) &= \int\limits_0^\infty p_{\alpha_{il}}(x)\frac{1}{2\pi}\int\limits_0^{2\pi} e^{j\nu x(\pm\cos\theta\ u \pm \sin\theta)} d\theta \\ &= \int\limits_0^\infty p_{\alpha_{il}}(x) J_0(\sqrt{1+u^2}\nu x) dx \\ &= \begin{cases} {}_1F_1\left(m_{il}; 1; \frac{-\Omega_{il}\nu^2}{4m_{il}}(1+u^2)\right) \\ \qquad \text{for Nakagami-}m\text{ fading} \\ \exp\left(\frac{-\Omega_{il}\nu^2}{4(1+K_{il})}(1+u^2)\right) J_0\left(\sqrt{\frac{2K_{il}\Omega_{il}}{1+K_{il}}}\nu\sqrt{1+u^2}\right), \\ \qquad \text{for Rician fading} \end{cases} \end{aligned} \tag{23}$$

and $\Phi^{(3)}_{y_{il}}(\nu) = \int\limits_0^1 \Phi^{(3)}_{y_{il}}(\nu|u) du$, which can be solved easily by using numerical methods.

Hence, the final expression for the CF of y_{il} can be written as

$$\Phi_{y_{il}}(\nu) = \frac{1}{4}\Phi^{(1)}_{y_{il}}(\nu) + \frac{1}{4}\Phi^{(2)}_{y_{il}}(\nu) + \frac{1}{2}\Phi^{(3)}_{y_{il}}(\nu). \tag{24}$$

Note that when the interfering signals are symbol synchronized with the desired symbol, then from (17), $\lambda_4 = c_0\cos\hat{\phi}_{il} + d_0\sin\hat{\phi}_{il}$ which is statistically equivalent to $\lambda_4 = \cos\hat{\phi}_{il} + \sin\hat{\phi}_{il}$ since c_0 and d_0 are independent Bernoulli RVs and will not change the statistics of $\cos\hat{\phi}_{il}$ or $\sin\hat{\phi}_{il}$. Thus, in this case, $\Phi_{y_{il}}(\nu)$ is equal to $\Phi^{(1)}_{y_{il}}(\nu)$ as given by (20). Finally, it is worthful to mention that although we consider NRZ signaling pulse in this study, the generality of the proposed analysis framework make the results applicable and could be extended to any suggested pulse shaping filtering.

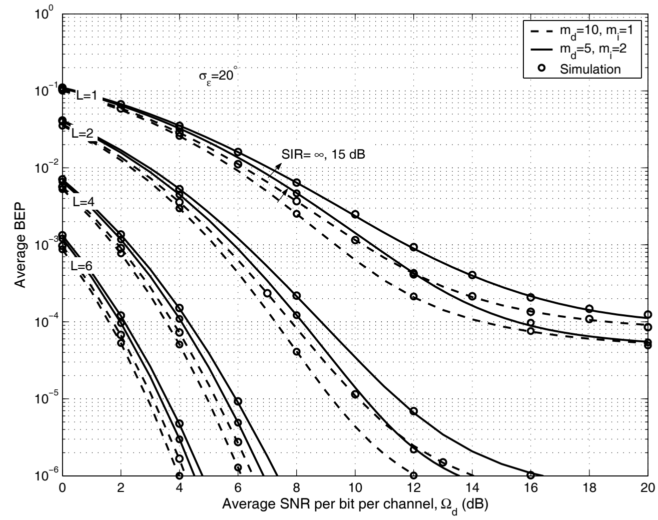

Fig. 1. Average BEP of partially coherent BPSK in EGC with Nakagami-m desired signal and a single asynchronous Nakagami-m interferer.

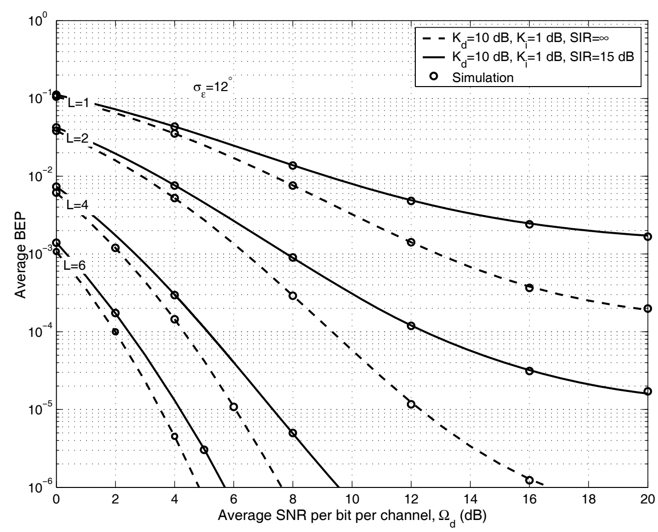

Fig. 2. Average BEP of partially coherent QPSK in EGC with Rician desired signal and a single synchronized Rician interferer.

IV. Numerical Results and Discussion

In all results presented in this section, and without lack of generality, we consider identical fading channels for both the desired user and the interfering signals, i.e., $\{K_{(d,i)l}, m_{(d,i)l}, \Omega_{(d,i)l}, \sigma_{\epsilon l}\} = \{K_{(d,i)}, m_{(d,i)}, \Omega_{(d,i)}, \sigma_\epsilon\}$ for all $l = 1, 2, \cdots, L$. This implies that the total interference power is uniformly distributed among all the interferers, i.e., $\sum_{i=1}^{I} P_b\Omega_{il} = IP_b\Omega$, where $P_b\Omega$ is the average received power of each interfering signal.

Figs. 1 and 2 show the average BEP versus the desired user's average SNR per bit of one diversity channel, Ω_d. Fig. 1 is for

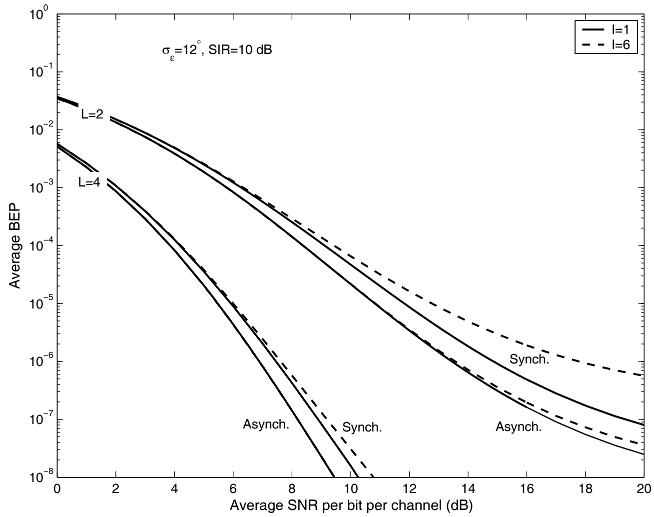

Fig. 3. Average BEP of partially coherent BPSK in EGC with Nakagami-m desired signal ($m_d = 8$) and (1, 6) Nakagami-m interferers ($m_i = 2$) at SIR=10 dB and $\sigma_\epsilon = 12°$.

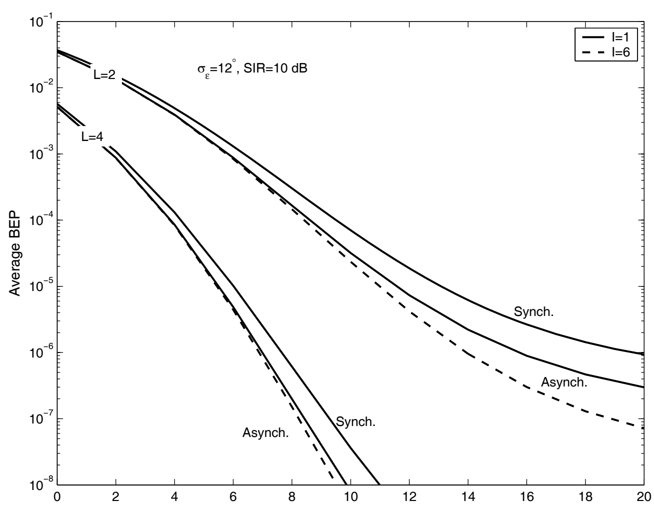

Fig. 4. Average BEP of partially coherent BPSK in EGC with Nakagami-m desired signal ($m_d = 8$) and (1, 6) synchronous/asynchronous Rayleigh interferers ($m_i = 1$) at SIR=10 dB and $\sigma_\epsilon = 12°$.

Nakagami-m/Nakagami-m desired/interfering BPSK signals with a single ($I = 1$) asynchronous interferer. The result for QPSK system is given in Fig. 2 for Rician/Rician model for the case where a single synchronized interferer (i.e., no cross-signal ISI) is assumed. The results are parameterized on the order of diversity, L, the system SIR, $\Omega_d/I\Omega_i$, and the fading severity conditions, i.e., different (m_d, m_i) or (K_d, K_i) fading scenarios. Quazi-analytical simulation (see [11] for discussion details) results are also given and they tightly agree with the theoretical results, which support the accuracy of the analytical results presented.

As a matter of fact, the results show that the average BEP performance improves (BEP decreases) when the desired signal is less severely faded, the order of diversity combining increases, or as the SIR increases. Note that when SIR=∞, the system does not suffer from CCI, which represents the lower bound on the system performance. We observe here that the irreducible error floor (shows clearly better in the succeeding figures) occurs now due to both the phase errors and the CCI signals which allow the decision statistic to take randomly both positive and negative quantities even in the absence of the AWGN (as SNR$\rightarrow \infty$). Although both carrier phase error and CCI will degrade the error rate performance, our attempt here is to investigate the impact of the CCI on the system performance since the effect of carrier error recovery has been studied in [1].

To see how the distribution of a given amount of interference power among a certain number of interferers will affect the performance, we plot in Figs. 3 and 4 the error rates for BPSK when the desired signal is corrupted by either one or six interferers under both synchronous and asynchronous symbol timing conditions. Fig. 3 shows that for both synchronous and asynchronous operations the average BEP when all interference power is concentrated in a single interferer is smaller (becomes more apparent for high SNR) than that obtained when the total interference power is equally distributed among six interferes. This observation agrees with the finding in [4, Fig. 1] where perfect coherent non diversity BPSK system was studied. On the other hand, Fig. 4 shows that for asynchronous Rayleigh-faded interferers, the BEP is better when the interference power is distributed among six interferers than that if it is concentrated in a single interferer. This finding consists with the one in [3, Fig. 4] for the case of perfect coherent Nakagami-Rayleigh channel. From the above discussion, one can draw the conclusion that the relationship between the average BEP and the distribution of the total interference power among a number of interferers has no clear cut edge. In some cases, the BEP is better when the interference power is concentrated in a single interferer, while, in other cases, its better if the interference power is equally distributed among more than one interferers. The performance comparison basically relies on the fading severity level of the interfering signals as well as wether those signals are symbol timing-synchronized with the desired user's signal or not.

It is also worthy to note from Fig. 4 that for synchronized interfering Rayleigh signals the BEPs are exactly the same disregarding the number of interferers. It is because for synchronized Rayleigh interferers, the CCI component z_{il} in (7) is Gaussian-distributed regardless of the number of interferers I, and hence, the associated BEP depends on the total interference power. A quick comparison between Figs. 3 and 4 shows that when the interfering signals are lightly faded (i.e., for larger values of m_i), the error rate performance improves. For example for six asynchronous interferers at $L = 2$ and SNR of 20 dB, the error rate when $m_i = 1$ is 7.6×10^{-8} while it is about 4×10^{-8} for $m_i = 2$. Such observation may look inconvenient at the first glance but it agrees with the results obtained [3], where a single channel perfectly coherent BPSK system with CCI is analyzed.

Finally, Fig. 5 shows the error rate performance as a function of the SIR. One can depict that as the SIR decreases, the system performs worst and the average BEP increases, as expected. To be a bit more specific, Table I displays the SNR gain penalty for BPSK Nakagami-m/Nakagami-m CCI system at BEP of 10^{-3} for several SIR values. Also, the gain penalties for Rician/Rician QPSK system are given in Table II. For

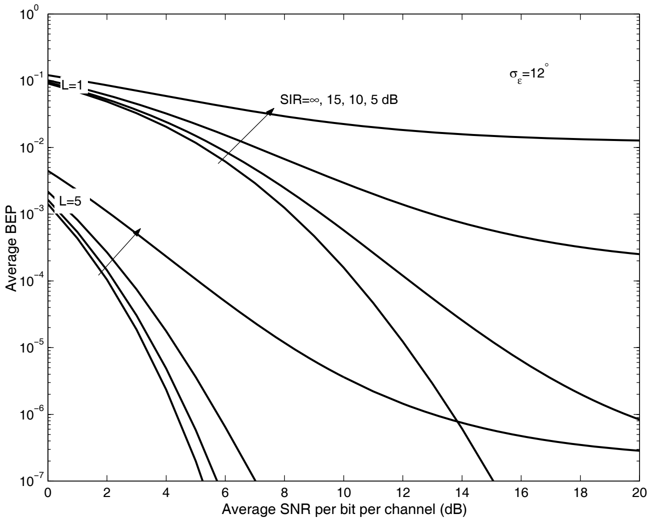

Fig. 5. Average BEP of partially coherent BPSK in EGC with Nakagami-m desired signal ($m_d = 10$) and a single synchronized Rayleigh interferer ($m_i = 1$) for different SIR levels and at $\sigma_\epsilon = 12°$.

TABLE I
GAIN PENALTY (IN DB) AT A BEP 10^{-3} DUE TO CCI WITH VARIOUS SIR VALUES (IN DB). THE SYSTEM IS NAKAGAMI-m_d/NAKAGAMI-m_i PARTIALLY COHERENT BPSK ($\sigma_\epsilon = 12°$) WITH SINGLE SYNCHRONIZED INTERFERER AND L-BRANCH EGC

	$m_d = 10,\ m_i = 1$				$m_d = 5,\ m_i = 2$			
L	SIR=25	20	15	10	SIR=25	20	15	10
1	0.10	0.30	1.0	4.80	0.13	0.41	1.35	6.0
2	0.05	0.13	0.42	1.50	0.05	0.14	0.48	1.70
3	0.03	0.08	0.27	0.90	0.03	0.10	0.31	1.0
4	0.01	0.06	0.19	0.68	0.02	0.07	0.22	0.70

Nakagami-m fading systems, we evaluate the gain penalties for two fading severity scenarios: ($m_d = 10,\ m_i = 1$) and ($m_d = 5,\ m_i = 2$). Similarly, the results for Rician fading systems are given for ($K_d = 10,\ K_i = 2$) dBs and ($K_d = 7,\ K_i = 3$) dBs. As we can see from those tables, the first scenario will result in less gain penalties for all SIR levels. Also, we can readily note that the gain penalty decreases (i.e., CCI is better opposed) as the system diversity order, L, increases. Hence, a general conclusion one can derive here is that the diversity reception has the ability of reducing the negative effect of CCI on the system performance besides its ability to combat fading and carrier phase error.

V. CONCLUSION

In this paper, we studied the error rate performance of partially coherent PSK diversity systems with CCI and cross-signal ISI. The results have shown that the desired user error rate performance with asynchronous interfering signals is better than the performance when the interfering signals are timing-synchronized with the desired signal. Also, it was seen that for a fixed desired user's fading conditions, the BEP improves as the interfering signals undergo lightly fading. Depending on the fading severity level, as well as, the timing synchronism of the interfering signals, it has been recognized that the system BEP performance is better when the interference power is concentrated in a single interferer in some cases, while in other cases, the BEP is better if the interference power is equally distributed among more than one interferers. Finally, the numerical results show that systems with diversity combining have the ability of compensating for the degradation caused by CCI.

TABLE II
GAIN PENALTY (IN DB) AT A BEP 10^{-3} DUE TO CCI WITH VARIOUS SIR VALUES (IN DB). THE SYSTEM IS RICIAN-K_d/RICIAN-K_i PARTIALLY COHERENT QPSK ($\sigma_\epsilon = 9°$) WITH SINGLE SYNCHRONIZED INTERFERER AND L-BRANCH EGC

	$K_d = 10,\ K_i = 2$ dBs				$K_d = 7,\ K_i = 3$ dBs			
L	SIR=25	20	15	10	SIR=25	20	15	10
1	0.35	1.15	5.25	∞	0.75	2.90	∞	∞
2	0.10	0.34	1.16	5.49	0.14	0.45	1.60	12.0
3	0.07	0.22	0.66	2.45	0.07	0.23	0.78	3.13
4	0.03	0.14	0.45	1.58	0.04	0.15	0.51	1.86

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